Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 6

Series

251. 9709_m22_qp_12 Q: 3

Find the term independent of x in each of the following expansions.

(a) $\left(3x + \dfrac{2}{x^2}\right)^6$ [3]

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(b) $\left(3x + \dfrac{2}{x^2}\right)^6 (1 - x^3)$ [3]

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252. 9709_m22_qp_12 Q: 4

The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

253. 9709_m21_qp_12 Q: 1

(a) Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

254. 9709_m21_qp_12 Q: 9

The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

255. 9709_s21_qp_11 Q: 2

The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

256. 9709_s21_qp_11 Q: 3

(a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x. [3]

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(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

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257. 9709_s21_qp_11 Q: 5

The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

258. 9709_s21_qp_12 Q: 4

The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

259. 9709_s21_qp_12 Q: 8

The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b+3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

260. 9709_s21_qp_13 Q: 7

(a) Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

261. 9709_s21_qp_13 Q: 9

(a) A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

262. 9709_w21_qp_11 Q: 1

(a) Expand $\left(1 - \frac{1}{2x}\right)^2$. [1]

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(b) Find the first four terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

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(c) Hence find the coefficient of x in the expansion of $\left(1 - \frac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

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263. 9709_w21_qp_11 Q: 4

The first term of an arithmetic progression is a and the common difference is -4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

264. 9709_w21_qp_12 Q: 5

The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

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(b) Hence find the exact sum of the first 25 terms of the progression. [3]

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265. 9709_w21_qp_12 Q: 6

The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

266. 9709_w21_qp_12 Q: 8

(a) It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

(b) It is given instead that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

267. 9709_w21_qp_13 Q: 2

(a) Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

268. 9709_w21_qp_13 Q: 4

The first term of an arithmetic progression is 84 and the common difference is -3.

(a) Find the smallest value of n for which the nth term is negative. [2]

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

269. 9709_m20_qp_12 Q: 6

The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

270. 9709_m20_qp_12 Q: 8

A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

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At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

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271. 9709_s20_qp_11 Q: 1

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

272. 9709_s20_qp_11 Q: 2

The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

273. 9709_s20_qp_11 Q: 3

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was $36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

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(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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274. 9709_s20_qp_12 Q: 1

(a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

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(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

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275. 9709_s20_qp_12 Q: 4

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

276. 9709_s20_qp_13 Q: 4

(a) Expand $(1+a)^5$ in ascending powers of a up to and including the term in a^3. [1]

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(b) Hence expand $[1+(x+x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

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277. 9709_s20_qp_13 Q: 8

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

278. 9709_w20_qp_11 Q: 5

In the expansion of $\left(2x^2 + \dfrac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \dfrac{a}{x}\right)^6$. [1]

279. 9709_w20_qp_11 Q: 8

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

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It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

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280. 9709_w20_qp_12 Q: 1

The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

281. 9709_w20_qp_12 Q: 2

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

282. 9709_w20_qp_12 Q: 4

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

283. 9709_w20_qp_13 Q: 5

In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

284. 9709_w20_qp_13 Q: 7

The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

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(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

285. 9709_m19_qp_12 Q: 1

The coefficient of x^3 in the expansion of $(1 - px)^5$ is -2160. Find the value of the constant p. [3]

286. 9709_m19_qp_12 Q: 6

(i) The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

287. 9709_s19_qp_11 Q: 1

The term independent of x in the expansion of $\left(2x+\frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

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(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

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288. 9709_s19_qp_11 Q: 8

(a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

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(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

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Scheme B ....

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289. 9709_s19_qp_12 Q: 1

Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

290. 9709_s19_qp_12 Q: 10

(a) In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

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(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

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(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

291. 9709_s19_qp_13 Q: 2

(i) In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

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(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

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292. 9709_s19_qp_13 Q: 5

Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

(i) Write down an expression for his total weight loss after x weeks. [1]

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(ii) He reaches his 13 kg target during week n. Use your answer to part **(i)** to find the value of n. [2]

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Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

293. 9709_w19_qp_11 Q: 1

Find the term independent of x in the expansion of $\left(2x + \dfrac{1}{4x^2}\right)^6$. [3]

294. 9709_w19_qp_11 Q: 4

A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

(ii) Find the total distance she runs over the 21 days. [2]

295. 9709_w19_qp_12 Q: 1

The coefficient of x^2 in the expansion of $(4+ax)\left(1+\frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

296. 9709_w19_qp_12 Q: 8

(a) Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

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(ii) Find the total distance she runs in the 21-day period. [2]

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(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

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(ii) Find the fourth term of the progression. [2]

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(iii) Find the sum to infinity of the progression. [2]

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297. 9709_w19_qp_13 Q: 1

298. 9709_w19_qp_13 Q: 9

The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

299. 9709_m18_qp_12 Q: 2

(i) Find the coefficients of x^2 and x^3 in the expansion of $(1-2x)^7$. [3]

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(ii) Hence find the coefficient of x^3 in the expansion of $(2+5x)(1-2x)^7$. [2]

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300. 9709_m18_qp_12 Q: 3

On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day.
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day.

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

301. 9709_s18_qp_11 Q: 1

(i) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

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(ii) Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant a. [3]

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302. 9709_s18_qp_11 Q: 8

(a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

303. 9709_s18_qp_12 Q: 1

The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

304. 9709_s18_qp_12 Q: 3

A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

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(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

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305. 9709_s18_qp_13 Q: 2

Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

306. 9709_s18_qp_13 Q: 3

The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

307. 9709_w18_qp_11 Q: 4

The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

308. 9709_w18_qp_12 Q: 1

Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

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309. 9709_w18_qp_12 Q: 5

The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

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(ii) Find the fourth term of each progression. [3]

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310. 9709_w18_qp_13 Q: 1

Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$. [3]

311. 9709_w18_qp_13 Q: 5

In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a. [6]

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312. 9709_m17_qp_12 Q: 2

In the expansion of $\left(\frac{1}{ax} + 2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

313. 9709_s17_qp_11 Q: 1

The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

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314. 9709_s17_qp_11 Q: 4

(a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

315. 9709_s17_qp_12 Q: 1

(i) Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

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(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

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316. 9709_s17_qp_12 Q: 7

(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

317. 9709_s17_qp_13 Q: 1

The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a. [3]

318. 9709_s17_qp_13 Q: 2

The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take. [2]

319. 9709_w17_qp_11 Q: 3

(a) A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

320. 9709_w17_qp_12 Q: 1

Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

321. 9709_w17_qp_12 Q: 3

(a) Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

322. 9709_w17_qp_13 Q: 1

An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

323. 9709_w17_qp_13 Q: 3

(i) Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

324. 9709_m16_qp_12 Q: 1

(i) Find the coefficients of x^4 and x^5 in the expansion of $(1 - 2x)^5$. [2]

(ii) It is given that, when $(1 + px)(1 - 2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

325. 9709_m16_qp_12 Q: 3

The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

326. 9709_s16_qp_11 Q: 1

Find the term independent of x in the expansion of $\left(x - \dfrac{3}{2x}\right)^6$. [3]

327. 9709_s16_qp_11 Q: 9

(a) The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

328. 9709_s16_qp_12 Q: 4

Find the term that is independent of x in the expansion of

(i) $\left(x - \frac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$. [4]

329. 9709_s16_qp_12 Q: 9

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

330. 9709_s16_qp_13 Q: 1

Find the coefficient of x in the expansion of $\left(\frac{1}{x} + 3x^2\right)^5$. [3]

331. 9709_s16_qp_13 Q: 4

The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

332. 9709_w16_qp_11 Q: 2

Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$. [4]

333. 9709_w16_qp_11 Q: 5

The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

334. 9709_w16_qp_12 Q: 4

In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

335. 9709_w16_qp_12 Q: 8

(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

336. 9709_w16_qp_13 Q: 2

The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a. [4]

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337. 9709_w16_qp_13 Q: 9

(a) Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

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338. 9709_s15_qp_11 Q: 3

(i) Find the first three terms, in ascending powers of x, in the expansion of

(a) $(1 - x)^6$, [2]

(b) $(1 + 2x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $[(1 - x)(1 + 2x)]^6$. [3]

339. 9709_s15_qp_11 Q: 7

(a) The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

340. 9709_s15_qp_12 Q: 3

(i) Find the coefficients of x^2 and x^3 in the expansion of $(2 - x)^6$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(3x + 1)(2 - x)^6$. [2]

341. 9709_s15_qp_12 Q: 7

The point C lies on the perpendicular bisector of the line joining the points $A\,(4,\ 6)$ and $B\,(10,\ 2)$. C also lies on the line parallel to AB through $(3,\ 11)$.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

342. 9709_s15_qp_13 Q: 3

(i) Write down the first 4 terms, in ascending powers of x, of the expansion of $(a - x)^5$. [2]

(ii) The coefficient of x^3 in the expansion of $(1 - ax)(a - x)^5$ is -200. Find the possible values of the constant a. [4]

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343. 9709_s15_qp_13 Q: 9

(a) The first term of an arithmetic progression is -2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

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344. 9709_w15_qp_11 Q: 1

In the expansion of $\left(1 - \dfrac{2x}{a}\right)(a + x)^5$, where a is a non-zero constant, show that the coefficient of x^2 is zero. [3]

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345. 9709_w15_qp_11 Q: 8

The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

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346. 9709_w15_qp_12 Q: 2

In the expansion of $(x + 2k)^7$, where k is a non-zero constant, the coefficients of x^4 and x^5 are equal. Find the value of k. [4]

347. 9709_w15_qp_13 Q: 2

Find the coefficient of x in the expansion of $\left(\frac{x}{3} + \frac{9}{x^2}\right)^7$. [4]

348. 9709_w15_qp_13 Q: 6

A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]